

Alpha Decay Chains of Superheavy Nuclei ²⁷⁸⁻²⁸²Rg

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Abstract: The alpha decay half lives and decay modes of five experimentally synthesized isotopes of superheavy nuclei roentgenium within the range $278 \leq A \leq 282$ have been studied using the Coulomb and proximity potential model for deformed nuclei (CPPMDN). By comparing the alpha decay half-lives with the corresponding spontaneous fission half-lives we have predicted 3 α chains from ^{278-280,282}Rg. The isotope ²⁸¹Rg is predicted to decay through spontaneous fission. The theoretical predictions are compared with experimental results and are seen to be matching well. For a theoretical comparison the alpha decay half-lives are also calculated using the Viola-Seaborg semi-empirical relationship, Universal formula of Poenaru et al., the analytical formula of Royer and the Universal decay law. The spontaneous fission half-lives are calculated using the new shell-effect-dependent formula proposed by Santhosh et al., and the semi-empirical formula of Xu et al. The predictability of our model, CPPMDN, in superheavy region is evident from the study.

Keywords: Spontaneous fission, Alpha radioactivity, Superheavy nuclei, Half life.

I. Introduction

The study of superheavy nuclei is one of the hottest topics in the current nuclear physics. Elements with $Z \geq 104$ are usually referred to as superheavy elements. The search for superheavy nuclei has gained its ambience since the prediction of magic island or island of stability [1, 2]. In the 1960s, Sobiczewski and his collaborators [3] had predicted new nuclear shell closures by studying the nuclear structure of light nuclei [4]. Around the same time, two groups, Myers and Swiatecki [1] and Viola and Seaborg [5] independently predicted the existence of island of stability where the superheavy nuclei will exist.

Two types of fusion evaporation reactions, namely the cold fusion reaction [6] and the hot fusion reaction [7] are used for the synthesis of superheavy nuclei. The use of cold fusion reactions enabled the discovery of six new elements, from bohrium ($Z=107$) to nihonium ($Z=113$) [8] and the use of hot fusion reactions led to the discovery of elements up to oganesson ($Z=118$) [8].

Superheavy nuclei decay mainly by the emission of α particles followed by subsequent spontaneous fission (SF). So the identification of new nuclides can be achieved by studying their characteristic alpha decay chains. Several theoretical works [9-12] have been performed in order to understand the formation of superheavy nuclei and their alpha decay half-lives. Also a number of theoretical studies [13-16] have been done for explaining the phenomenon of spontaneous fission in superheavy nuclei.

The intension of our present work is to compare the decay modes and half-lives of five experimentally detected isotopes of roentgenium ($Z = 111$) with our theoretical predictions. The discovery of Rg was first reported in 1994 by Hofmann et al., [17] by detecting three events of α decay chains from ²⁷²Rg. The isotopes ²⁷⁸⁻²⁸²Rg were reported by Oganessian et al., from the decay chains of ²⁸²Nh, ^{287,288}Mc, ^{293,294}Ts [7] respectively. In the present paper, the α decay half-lives and decay modes of ²⁷⁸⁻²⁸²Rg has been studied within the Coulomb and proximity potential model for deformed nuclei (CPPMDN) [18], which is an extension of Coulomb and proximity potential model (CPPM) [19] proposed by Santhosh et al. The matching between the experimental and theoretical results suggests the predictability of the model in the superheavy region.

The overview of the paper is as follows: In Section 2 a brief description of CPPMDN is given. The results and discussion on the alpha decay properties of the selected isotopes are presented in Section 3 and the last section summarizes the entire work.

II. Coulomb and proximity potential model for deformed nuclei (CPPMDN)

In CPPMDN the interacting potential between two nuclei is taken as the sum of deformed Coulomb potential, deformed two term proximity potential and centrifugal potential, for both the touching configuration and for the separated fragments. For the pre-scission (overlap) region, simple power law interpolation has been used.

The interacting potential barrier for two spherical nuclei is given by:

$$V = \frac{Z_1 Z_2 e^2}{r} + V_p(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \text{ for } z > 0 \quad (1)$$

Here Z_1 and Z_2 are the atomic numbers of the daughter and emitted cluster, ‘ r ’ is the distance between fragment centres, ‘ z ’ is the distance between the near surfaces of the fragments, ℓ represents the angular momentum and μ the reduced mass. V_p is the proximity potential given by Blocki et al., [20, 21] as:

$$V_p(z) = 4\pi\gamma b \left[\frac{C_1 C_2}{(C_1 + C_2)} \right] \Phi\left(\frac{z}{b}\right) \quad (2)$$

With the nuclear surface tension coefficient:

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2 / A^2] \text{ MeV/fm}^2 \quad (3)$$

Here N , Z and A represent the neutron, proton and mass number of the parent nuclei. Φ represents the universal proximity potential [21] given as:

$$\Phi(\varepsilon) = -4.41e^{-\varepsilon/0.7176} \text{ for } \varepsilon > 1.9475 \quad (4)$$

$$\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3 \text{ for } 0 \leq \varepsilon \leq 1.9475 \quad (5)$$

With $\varepsilon = z/b$, where $b \approx 1$ fm is the width (diffuseness) of the nuclear surface. The Süsmann central radii C_i of the fragments are related to the sharp radii R_i as:

$$C_i = R_i - \left(\frac{b^2}{R_i}\right) \text{ fm} \quad (6)$$

For R_i , we use semi-empirical formula in terms of mass number A_i as [20]:

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \text{ fm} \quad (7)$$

The potential for the internal part (overlap region) of the barrier is given as:

$$V = a_0(L - L_0)^n \text{ for } z < 0 \quad (8)$$

Where $L = z + 2C_1 + 2C_2$ fm and $L_0 = 2C$ fm, the diameter of the parent nuclei. The constants a_0 and n are determined by the smooth matching of the two potentials at the touching point.

The barrier penetrability P using the one dimensional Wentzel-Kramers-Brillouin approximation, is given as:

$$P = \exp\left\{-\frac{2}{\hbar} \int_a^b \sqrt{2\mu(V - Q)} dz\right\} \quad (9)$$

here the mass parameter is replaced by $\mu = mA_1A_2/A$, where m is the nucleon mass and A_1 , A_2 are the mass numbers of daughter and emitted cluster respectively. The turning points ‘ a ’ and ‘ b ’ are determined from the equation, $V(a) = V(b) = Q$, where Q is the energy released.

The half-life time is given by:

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right) \quad (10)$$

Here λ is the decay constant and ν is the assault frequency. The empirical vibration energy E_v , is given as [9]:

$$E_v = Q \left\{ 0.056 + 0.039 \exp\left[\frac{(4 - A_2)}{2.5}\right] \right\} \text{ for } A_2 \geq 4 \quad (11)$$

The Coulomb interaction between the two deformed and oriented nuclei taken from Ref. [22] with higher multipole deformations included [23, 24] is given as:

$$V_C = \frac{Z_1 Z_2 e^2}{r} + 3Z_1 Z_2 e^2 \sum_{\lambda, i=1,2} \frac{1}{2\lambda + 1} \frac{R_{0i}^\lambda}{r^{\lambda+1}} Y_\lambda^{(0)}(\alpha_i) \left[\beta_{\lambda i} + \frac{4}{7} \beta_{\lambda i}^2 Y_\lambda^{(0)}(\alpha_i) \delta_{\lambda,2} \right] \quad (12)$$

with:

$$R_i(\alpha_i) = R_{0i} \left[1 + \sum_{\lambda} \beta_{\lambda i} Y_\lambda^{(0)}(\alpha_i) \right] \quad (13)$$

Where $R_{0i} = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$. Here α_i is the angle between the radius vector and symmetry axis of the i^{th} nuclei (see Fig.1 of Ref [23]) and it is to be noted that the quadrupole interaction term proportional to $\beta_{21}\beta_{22}$, is neglected because of its short-range character.

The two-term proximity potential for interaction between a deformed and spherical nucleus is given by Baltz et al., [25] as:

$$V_{p2}(R, \theta) = 2\pi \left[\frac{R_1(\alpha)R_c}{R_1(\alpha) + R_c + S} \right]^{1/2} \left[\frac{R_2(\alpha)R_c}{R_2(\alpha) + R_c + S} \right]^{1/2} \times \left[\left[\varepsilon_0(S) + \frac{R_1(\alpha) + R_c}{2R_1(\alpha)R_c} \varepsilon_1(S) \right] \left[\varepsilon_0(S) + \frac{R_2(\alpha) + R_c}{2R_2(\alpha)R_c} \varepsilon_1(S) \right] \right]^{1/2} \quad (14)$$

Where θ is the angle between the symmetry axis of the deformed nuclei and the line joining the centers of the two interacting nuclei, and α corresponds to the angle between the radius vector and symmetry axis of the nuclei (see Fig. 5 of Ref [25]). $R_1(\alpha)$ and $R_2(\alpha)$ are the principal radii of curvature of the daughter nuclei, R_c is the radius of the spherical cluster, S is the distance between the surfaces along the straight line connecting the fragments, and $\varepsilon_0(S)$ and $\varepsilon_1(S)$ are the one dimensional slab-on-slab function.

III. Results and Discussion

The decay modes and half-lives of the superheavy nuclei Rg within the range $278 \leq A \leq 282$ has been studied in the present work. In order to find the decay modes, the α half-lives of the isotopes were compared with the corresponding Spontaneous fission half-lives. Those nuclei with α decay half-lives less than spontaneous fission half-lives will survive fission and hence decay through alpha emission. In the study, α decay half-lives calculated using CPPMDN [18] has been compared with the spontaneous fission half-lives calculated using the new shell-effect-dependent formula of Santhosh et al., [16] for predicting the decay modes. For a theoretical comparison, α decay half-lives were evaluated with five other models. A comparison between experimental and theoretical results [7] was also performed. In addition to the formula proposed by Santhosh et al., [16] the semi-empirical formula proposed by Xu et al., [14] has also been used for calculating the SF half-lives.

1.1. Alpha Decay Half-Lives

The key quantities in determining alpha decay half-lives are the Q value. In the present paper Q values are calculated using the mass excess values taken from the experimental mass table of Wang et al [26]. The electron screening effect [27, 28] on the energy of alpha particle is also incorporated while calculating the Q value.

Many phenomenological formulae are available for calculating alpha decay half-lives. In addition to CPPMDN we have used the Coulomb and proximity potential model (CPPM) [18], Viola-Seaborg semi-empirical relation (VSS) [5, 29], Universal curve of Poenaru et al., [30, 31], analytical formula of Royer [32] and the Universal decay law [33, 34] for calculating the alpha decay half-lives.

1.2. Spontaneous Fission Half-Lives

The Spontaneous fission half-lives were computed using the new shell-effect-dependent formula of Santhosh et al., [16] and is given by:

$$\log_{10}(T_{1/2} / yr) = a \frac{Z^2}{A} + b \left(\frac{Z^2}{A} \right)^2 + c \left(\frac{N-Z}{N+Z} \right) + d \left(\frac{N-Z}{N+Z} \right)^2 + e E_{shell} + f \quad (15)$$

Where $a = -43.25203$, $b = 0.49192$, $c = 3674.3927$, $d = -9360.6$, $e = 0.8930$ and $f = 578.56058$. E_{shell} is the shell correction energy taken from Ref. [35].

For a theoretical comparison, the spontaneous fission half-lives were also evaluated using the semi-empirical formula proposed by Xu et al [14]. Due to the complexity in the process of fission and uncertainty in the nature of fission barrier, accurate calculation of spontaneous fission half-lives are difficult. Hence model to model variations can be seen while calculating the fission half-lives. The comparison of alpha decay half-lives with the spontaneous fission half-lives calculated within our model and the predictions on the decay chains are presented in TABLE 1. The comparison of the present values with other two theoretical models and with the experimental results [7] is also given.

Table 1: The comparison of the calculated alpha decay half-lives with the spontaneous fission half-lives for the isotopes ²⁷⁸⁻²⁸²Rg and its decay products.

Parent nuclei	Q_α (MeV)	T_{SF} (s)		$T_{1/2}^\alpha$ (s)				Expt. half life (s)	Mode of Decay	
		Xu	KPS	CPPMDN	CPPM	UNIV	Royer		Exp.[7]	Th.
²⁷⁸ Rg	10.90	3.47E-01	7.68E-01	6.15E-04	3.19E-03	3.27E-04	4.18E-03	4.20E-03	α	α
²⁷⁹ Mt	10.56	2.79E-02	1.26E+01	6.30E-04	5.57E-03	5.78E-04	6.84E-03	4.40E-01	α	α
²⁷⁹ Bh	9.12	3.54E-02	5.39E+01	7.14E-01	2.22E+01	1.11E+00	2.46E+01	6.10E+01	α	α
²⁸⁰ Db	8.27	7.69E-01	1.24E+02	3.69E+03	3.69E+03	1.32E+02	3.80E+03	1.32E+03	SF	SF
²⁷⁹ Rg	10.57	1.35E-01	5.94E-01	4.25E-03	2.39E-02	1.96E-03	8.13E-03	9.00E-02	α	α
²⁷⁹ Mt	10.27	9.49E-03	1.18E+00	4.06E-03	3.42E-02	2.91E-03	1.15E-02	2.00E-02	α	α
²⁷⁹ Bh	9.54	7.58E-03	7.80E+00	4.91E-02	9.67E-01	6.41E-02	2.64E-01	1.50E+00	α	α
²⁸⁰ Db	7.98	8.16E-02	4.81E+01	8.73E+02	4.68E+04	1.37E+03	6.44E+03	4.68E+03	SF	SF
²⁸⁰ Rg	10.25	7.08E-02	1.79E-01	4.18E-02	1.84E-01	1.22E-02	2.33E-01	4.60E+00	α	α
²⁷⁹ Mt	10.05	4.95E-03	1.91E-01	1.77E-02	1.42E-01	1.04E-02	1.68E-01	4.50E-01	α	α
²⁷⁹ Bh	9.36	3.93E-03	1.07E+00	1.73E-01	3.43E+00	2.01E-01	3.79E+00	1.09E+01	α	α
²⁸⁰ Db	8.31	4.16E-02	1.36E+01	4.61E+01	2.44E+03	8.85E+01	2.48E+03	9.36E+04	SF	SF
²⁸¹ Rg	9.82	6.39E-03	9.26E-02	5.76E-01	3.38E+00	1.68E-01	8.15E-01	1.70E+01	α :0.1 SF:0.9	SF
²⁷⁹ Mt	9.77	4.13E-04	1.11E-02	1.51E-01	9.33E-01	5.64E-02	2.47E-01	5.00E-03	SF	SF
²⁸² Rg	9.56	3.22E-03	1.40E+00 [†]	5.21E-01	2.13E+01	8.86E-01	2.62E+01	1.00E+02	α	α
²⁷⁹ Mt	9.52	2.08E-04	2.88E-02 [†]	1.53E-02	5.37E+00	2.74E-01	6.21E+00	4.50E+00	α	α
²⁷⁹ Bh	8.98	1.36E-04	1.54E+00 [†]	3.64E-01	5.66E+01	2.55E+00	6.12E+01	4.40E+01	α	α
²⁸⁰ Db	8.37	8.45E-04	1.42E+01 [†]	6.11E+01	1.36E+03	5.11E+01	1.37E+03	5.40E+04	SF	SF

[†] T_{SF} taken from Ref. [13].

From TABLE 1, it is seen that, by comparing the alpha decay half-lives calculated within CPPMDN with the spontaneous fission half-lives calculated using the shell-effect-dependent formula of Santhosh et al., 3α chains can be predicted from the isotopes ²⁷⁸⁻²⁸⁰Rg. The isotope ²⁸¹Rg is predicted to decay through spontaneous fission. In the case of ²⁸²Rg, for more accurate prediction on the decay modes we have used the spontaneous fission half-lives given by Smolanczuk et al [13]. It is evident that the theoretical predictions on the decay modes of all the isotopes of Rg within the range $278 \leq A \leq 286$ matches well with experimental results [7].

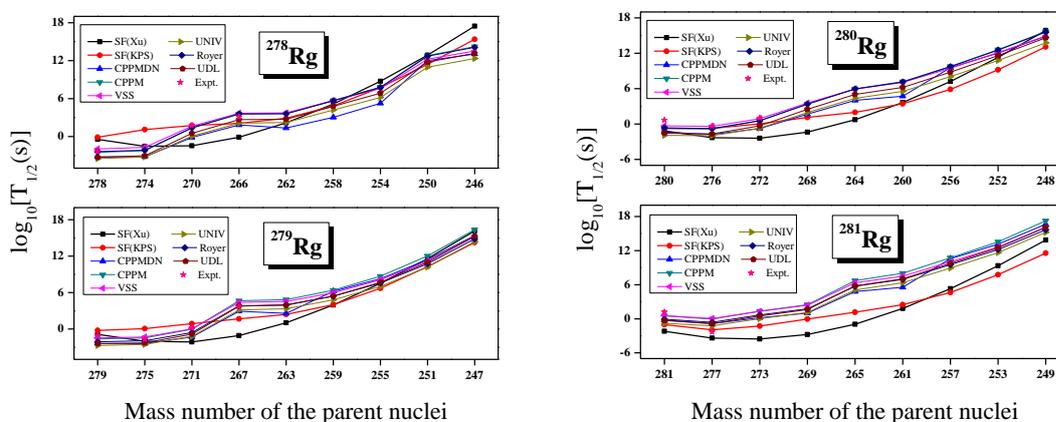
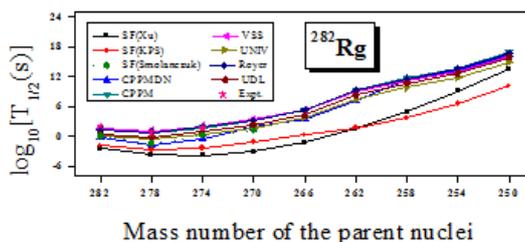

Fig 1: The comparison of the calculated alpha decay half-lives with the spontaneous fission half-lives for the isotopes ²⁷⁸⁻²⁸¹Rg.

Fig. 1 and 2 represent the plot for $\log_{10}T_{1/2}$ versus mass number for all the nuclei under study. All the calculations done within various theoretical models are shown.


Fig 2: The comparison of the calculated alpha decay half-lives with the spontaneous fission half-lives for the isotopes ²⁸²Rg.

IV. Conclusions

The studies of superheavy nuclei have prime importance in the field of nuclear physics. The confirmation of the region of island of stability and the understanding of how nuclei are held together can only be achieved through such study.

In the present study the alpha decay chains of isotopes of superheavy nuclei ²⁷⁸⁻²⁸²Rg are studied using CPPMDN. We have predicted 3α chains from the isotopes ^{278-280,282}Rg. The isotope ²⁸¹Rg decays through spontaneous fission. The obtained predictions are in good agreement with the experimental results. The alpha decay half-lives calculated using CPPMDN is compared with five other theoretical models. It is seen that for predicting the decay modes of superheavy nuclei CPPMDN suits better than the other models. As we are successful in reproducing the experimental results of ²⁷⁸⁻²⁸²Rg, we are planning to extend our work to predict the decay modes of all the isotopes of Rg within the range $259 \leq A \leq 339$. We hope that our studies will be a guide line for future experiments.

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